Guided Beam Waves Between Parallel Concave Reflectors

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Abstract—**A new guided beam wave transmission system is proposed here, which is composed of two parallel concave reflectors. The principle is a combination of waveguide and beam wave transmission. The shape of the reflector cross section and the corresponding mode functions were obtained. Attenuation due to wall current and limited aperture of the reflectors were calculated. Experiments were made to confirm the modes and the attenuation. One of the remarkable features of this transmission system is its field distribution, which is concentrated into a belt-shaped space between reflectors. Considering this feature, this system seems to be effectively applied to the railways as a medium for obstacle detection and communication.**

I. Introduction

SINCE G. Goubau et al. [1], [2] reported the beam wave transmission in 1961, there have been proposed several types of the structure conveying the beam modes. Circular dielectric lens was the original form of phase transformer [2] of the proposer. T. Nakahara proposed and analyzed a system of the iterative wing-shaped dielectric lenses between two parallel conductor plates [3]. The substitution of the concave reflectors for the dielectric lenses was independently proposed by M. Soejima and T. Nakahara [4] in 1963, and by J. E. Degenford et al. [5] in 1964. A gas lens system for optical trasmission was proposed and measurements were made by S. E. Miller et al. [6] and Beck [7].

All of these systems have a periodic structure in the propagation direction. The system proposed here has a uniform structure, as shown in Fig. 1 [8]. The waves are propagated, reflecting between reflectors just as in a waveguide and converged into a beam by concave reflectors on the transverse cross section.

According to analogy with behavior of the elementary waves in a rectangular waveguide mode, the modes in this system would be represented by a mixture of two elementary beam waves propagating in the direction that makes an angle θ with the z-axis in the z-x plane, as is shown in Fig. 2. The angle θ is approximately described by mode number m, which corresponds to a field change in the x-direction, spacing d of the reflectors, and free space wavelength λ as $\sin\theta = m\lambda/2d$, where m is an integer.

Thermal loss is presumed to be the same as that of the TE or TM mode propagation between two parallel conductor plates of the same spacing d.

Interval of the phase transformers for the elementary waves in this system is $d/\tan\theta$. Diffraction loss per a reflection can be obtained by substituting $d/\tan\theta$ for the phase transformer interval or the reflector spacing in the beam wave theory [1], [9]. This deduction shows that confocal setting of the reflections would be most favorable for low loss transmission. The analysis in this paper shows the validity of these physical observations.

Manuscript received April 18, 1966; revised August 23, 1966.
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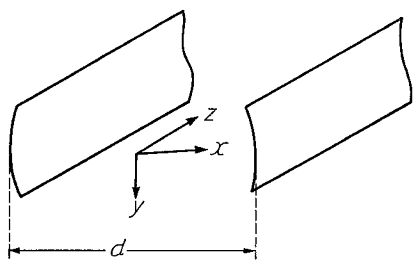

Fig. 1. Beam wave transmission system with parallel concave reflectors.

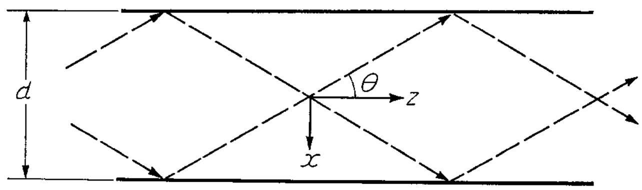

Fig. 2. Propagation of elementary beam waves.

For an analysis of beam waves, two main approaches have been presented. One applies beam mode description, which is approximated by the Gaussian-Laguerre function in the cylindrical coordinates [1], or by the Gaussian-Hermite function in the rectangular coordinates [3]. Another is the application of the Huygens principle [9], which leads to an integral equation for the field distribution on the reflectors with finite apertures. Although both approaches can be applied for this transmission system, analysis here is based on beam mode description in the rectangular coordinates.

One of the interesting aspects of this system is the similarity to the groove guide [10]–[12] in structure and in principle. The analysis here gives an approach to a particular case of the groove guide.

II. Reflector and Mode Function

When a field distribution is given of a beam mode, the cross-sectional surface of the reflectors that convey the beam mode can be obtained by considering the boundary condition on the surface. If the cross-sectional curve for each mode has similitude, mode functions for a given pair of reflectors can be obtained by the proper choice of the transverse wavenumbers for each mode.

The waves in a uniform waveguide, which has an isotropic, homogeneous, and lossless medium closed with a conductor surface, are classified into the TE and TM modes. Even if the transmission space is partially open, the classification may be true in a practical sense when the field distribution suffers a reasonably small change from the opening. In this sense, the mode functions in the following analysis are represented by a function Φ corresponding to the axial electric or magnetic field (E_z, H_z) as

$$\Phi = Ae^{\pm jhz}\int_{-\infty}^{\infty} f(\beta)e^{j\beta y}\begin{pmatrix}\cos k_x x\\ \sin k_x x\end{pmatrix} d\beta \tag{1}$$

$$E_x = \frac{-j}{k_t^2}\left(h\frac{\partial E_z}{\partial x} + \omega\mu_0\frac{\partial H_z}{\partial y}\right) \tag{2}$$

$$E_y = \frac{-j}{k_t^2}\left(h\frac{\partial E_z}{\partial y} - \omega\mu_0\frac{\partial H_z}{\partial x}\right) \tag{3}$$

$$H_x = \frac{j}{k_t^2}\left(\omega\epsilon_0\frac{\partial E_z}{\partial y} - h\frac{\partial H_z}{\partial x}\right) \tag{4}$$

$$H_y = \frac{-j}{k_t^2}\left(\omega\epsilon_0\frac{\partial E_z}{\partial x} + h\frac{\partial H_z}{\partial y}\right) \tag{5}$$

$$k_0^2 = k_t^2 + h^2 \tag{6}$$

$$k_t^2 = k_x^2 + \beta^2 \tag{7}$$

where

Φ = distribution function of E_z or H_z
ϵ_0, μ_0 = permittivity and permeability in free space
k_0 = wavenumber in free space
k_t = transverse wavenumber in the x-y plane
k_x = wavenumber in the x-direction
β = wavenumber in the y-direction
h = wavenumber in the z-direction
$f(\beta)$ = spectrum of the wavenumber β
A = amplitude constant.

On assumption that the spectrum function $f(\beta)$ has appreciable value only within a range $|\beta| < \beta_c$, where $\beta_c^2 \ll k_t^2$, and that $\beta_c^2 x/2k_t$ remains within the order of 2π, (1) can be described in a series of the Gaussian-Hermite functions [3],

$$\Phi = \frac{\exp\left(-\dfrac{W^2Y^2}{W^4+4X^2}\right)}{\sqrt[4]{W^4+4X^2}}\sum_{n=0}^{\infty} B_n He_n\left(\frac{2WY}{\sqrt{W^4+4X^2}}\right) \cdot\begin{pmatrix}\cos\\ \sin\end{pmatrix}\left[X + \frac{2XY^2}{W^4+4X^2} - \left(n+\frac{1}{2}\right)\cdot\tan^{-1}\frac{2X}{W^2}\right]\cdot e^{\pm jhz} \tag{8}$$

$$X = k_t x \tag{9}$$

$$Y = k_t y \tag{10}$$

$$W = \frac{k_t}{\beta_0} \tag{11}$$

where B_n's are arbitrary amplitude constants and β_0 is an arbitrary constant that corresponds to dispersion of the β-spectrum, as shown in preceding papers [1], [3].

The cross section of the reflectors that convey one of the modes represented by (8) is given approximately by

$$X + \frac{2XY^2}{W^4+4X^2} - \left(n+\frac{1}{2}\right)\tan^{-1}\frac{2X}{W^2} = \frac{m\pi}{2} \tag{12}$$

where m is an integer. The cross point (X_0, 0) of the reflector cross section to the X-axis is given for both TM and TE mode as

$$X_0 - \left(n+\frac{1}{2}\right)\tan^{-1}\frac{2X_0}{W^2} = \frac{m\pi}{2}. \tag{13}$$

On this condition, E_z of the TM modes is zero on the reflector surface, while the tangential component of the transverse electric field does not vanish on the surface. This contradiction comes from the fact that the field function is an approximate solution for the wave equation.

For the same reason, (13) does not satisfy rigorously the boundary condition for the TE modes. However, (13) corresponds to the phase front of the waves in the transverse plane, when the field distribution function Φ is expressed as a combination of the waves propagated in the X-direction.

$$\Phi = \frac{\exp\left(-\dfrac{W^2Y^2}{W^4+4X^2}\right)}{4\sqrt{W^4+4X^2}}\sum_{n=0}^{\infty} B_n He_n\left(\frac{2WY}{\sqrt{W^4+4X^2}}\right) \cdot\exp\begin{pmatrix}j\\ -j\end{pmatrix}\left[X + \frac{2XY^2}{W^4+4X^2} - \left(n+\frac{1}{2}\right)\cdot\tan^{-1}\frac{2X}{W^2}\right]\cdot e^{\pm jhz}. \tag{14}$$

The reflector surface should be placed along the phase front, so that the same field as the incident wave should be reflected if a mode is represented by only one term of the series in (8). This is why (12) gives the reflector for the TE modes.

When we confine the treatment on the reflector within the range where the reflector is nearly parallel to the Y-axis and the term $\tan^{-1}2X/W^2$ takes a small change in comparison with other terms, the cross section is given through (12) and (13) by

$$\left(\frac{Y}{X_0}\right)^2 = \frac{1}{2}\left(\frac{X_0}{X}-1\right)\left[\left(\frac{W^2}{X_0}\right)^2 + 4\left(\frac{X}{X_0}\right)^2\right]. \tag{15}$$

The equation is independent of the mode number m. This fact shows that the cross section preserves similitude for every mode, provided that W^2 takes a value proportional to X_0. Figure 3 shows examples of the reflector cross section. The radius of curvature R of the cross section on the X-axis is given by

$$R = X_0 + \frac{W^4}{4X_0^2} \tag{16}$$

$$R = k_t b \tag{17}$$

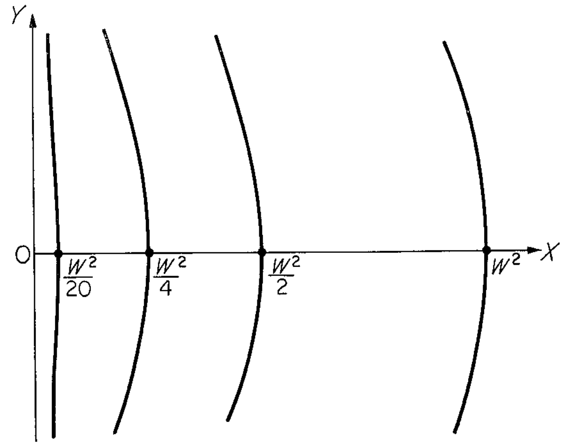

Fig. 3. Reflector cross section ($W=50$).

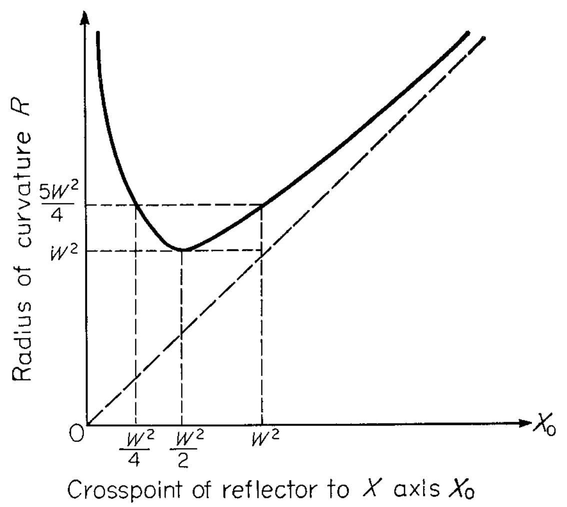

Fig. 4. Radius of curvature on x-axis.

where b is the physical measurement of the radius. Figure 4 shows this relationship.

The reflector cross section is given by (15) for a given mode. However, (8) should be reformed to represent the mode functions for a given pair of reflectors. The mode function for each mode can be obtained by suitable choice of the transverse wavenumber k_t and the dispersion factor of the β-spectrum β_0.

Here we treat only symmetrical setting of the reflectors to the y-z plane. Let the spacing of the reflector d and the radius of the curvature b on the x-axis be given, the transverse wavenumbers are determined by (9)–(11), (13), (16), and (17) as the following.

$$k_{mn} = \frac{1}{d}\left[m\pi + (2n+1)\tan^{-1}\frac{d}{\sqrt{2bd - d^2}}\right] \tag{18}$$

$$\beta_{mn} = \sqrt{\frac{k_{mn}}{\sqrt{2bd - d^2}}} \tag{19}$$

where k_{mn} and β_{mn} are, respectively, k_t and β_0 for the TM_{mn} or TE_{mn} mode. The subscript m denotes the number of the field change in the x-direction, and n in the y-direction.

Using k_{mn} and β_{mn}, the field distribution function Φ_{mn} for the TM_{mn} or the TE_{mn} modes are represented for a given pair of reflectors as

$$\Phi_{mn} = \frac{\exp\left\{\dfrac{-\beta_{mn}^2 y^2}{1 + 4\dfrac{\beta_{mn}^4 x^2}{k_{mn}^2}}\right\}}{\sqrt[4]{1 + 4\dfrac{\beta_{mn}^4 x^2}{k_{mn}^2}}} He_n\left\{\frac{2\beta_{mn} y}{\sqrt{1 + 4\dfrac{\beta_{mn}^4 x^2}{k_{mn}^2}}}\right\} \cdot \binom{\cos}{\sin}\left[k_{mn}x + \frac{2k_{mn}\beta_{mn}^4 x y^2}{k_{mn}^2 + 4\beta_{mn}^4 x^2} - \left(n + \frac{1}{2}\right) \cdot \tan^{-1}\frac{2\beta_{mn}^2 x}{k_{mn}}\right] \cdot e^{\pm j h_{mn} z} \tag{20}$$

where

$$k_0^2 = k_{mn}^2 + h_{mn}^2. \tag{21}$$

For confocal setting, (18)–(20) are reduced to

$$k_{mn} = \frac{\pi}{d}\left(m + n + \frac{1}{4}\right) \tag{22}$$

$$\beta_{mn} = \sqrt{\frac{k_{mn}}{d}} \tag{23}$$

$$\Phi_{mn} = \frac{\exp\left\{\dfrac{-k_{mn} y^2}{\left[1 + \left(\dfrac{2x}{d}\right)^2\right] d}\right\}}{\sqrt[4]{1 + \left(\dfrac{2x}{d}\right)^2}} He_n \cdot \left\{\frac{2\sqrt{k_{mn}}\, y}{\sqrt{\left[1 + \left(\dfrac{2x}{d}\right)^2\right] d}}\right\} \cdot \binom{\cos}{\sin}\left\{k_{mn}x + \frac{2k_{mn} x y^2}{\left[1 + \left(\dfrac{2x}{d}\right)^2\right] d^2} - \left(n + \frac{1}{2}\right)\tan^{-1}\frac{2x}{d}\right\} \cdot e^{\pm j h_{mn} z}. \tag{24}$$

Transverse field configuration near the z-axis and the corresponding reflector surfaces are shown in Fig. 5.

The spot size is defined as the value of y where the exponential part in the mode function takes the value $1/e$. The spot size w_0 on the y-axis and w_s on the reflector surface are given as,

$$w_0 = \frac{\sqrt[4]{2bd - d^2}}{\sqrt{k_{mn}}} \tag{25}$$

$$w_s = \frac{\sqrt{2d}}{\sqrt{k_{mn}}\,\sqrt[4]{\left(\dfrac{d}{b}\right)\left(2 - \dfrac{d}{b}\right)}}\,. \tag{26}$$

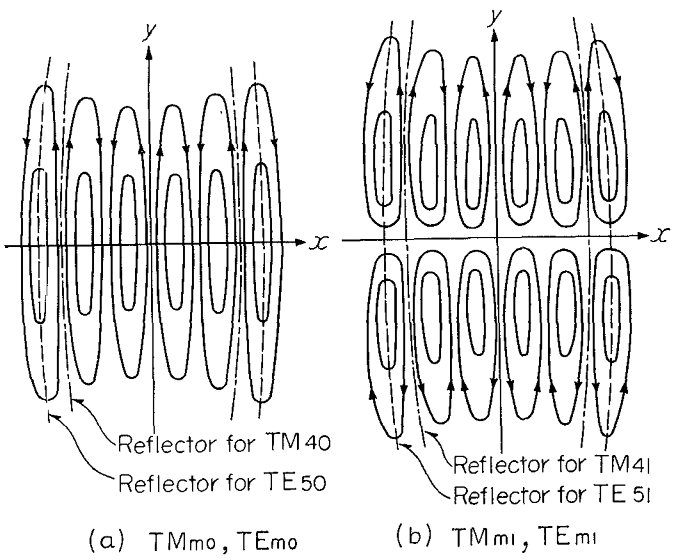

Fig. 5. Transverse electric (TE_{mn}) or magnetic (TM_{mn}) field near the z-axis.

This shows that the same discussion is made on the spot size as in the earlier papers [9], [13], provided that the free space wavenumber k_0 is replaced by the transverse wavenumber k_{mn}.

The propagation direction θ_{mn} of the elementary waves for the TM_{mn} or TE_{mn} mode as was shown in Fig. 2 is defined by the wavenumbers as

$$\theta_{mn} = \cos^{-1}\frac{h_{mn}}{k_0} = \sin^{-1}\frac{k_{mn}}{k_0} = \tan^{-1}\frac{k_{mn}}{h_{mn}}. \tag{27}$$

III. Attenuation

Transmission loss of this system consists of thermal loss and diffraction loss. Attenuation per unit length is given by the product of the attenuation for a single reflection and the number of reflections per unit length. Attenuation constant due to the thermal loss is given by the following equations, when the tangential component of the transverse magnetic field on the reflector surface is approximated by H_y.

$$\alpha_{tM} \simeq 2\frac{R_s k_0}{Z_0 h_{mn} d} = 2\frac{R_s}{Z_0 d\cos\theta_{mn}} \tag{28}$$

$$\alpha_{tE} \simeq 2\frac{R_s k_{mn}}{k_0 h_{mn} d}\left(1 + \frac{h_{mn}^2}{2k_{mn}^3 d}\right) = 2\frac{R_s\sin^2\theta_{mn}}{Z_0 d\cos\theta_{mn}}\left(1 + \frac{\cos^2\theta_{mn}}{2k_{mn}d\sin^2\theta_{mn}}\right) \tag{29}$$

where α_{tM} is the attenuation constant for the TM modes, and α_{tE} for the TE modes. Equation (28) and the first term in (29) coincide with the attenuation constant of the TM and TE modes between parallel plane plates. The second term in (29) is the attenuation due to the z-component of the current on the reflectors.

The diffraction loss for a single reflection at the finite aperture of $2a$ can be obtained by deriving an integral equation for the beam waves traveling in the x-direction. The derivation process follows closely the one applied in earlier papers [1], [3]. The integral equation for a pair of reflectors with a radius of curvature b and a spacing d is

$$F(\sigma, t) = \frac{e^{j(\pi/4 - k_t d)}}{P}\int_0^T F(\sigma, \tau)\sqrt{\frac{2}{\pi}} \cdot e^{j\tau^2(\sigma-1)}\begin{bmatrix}\cos(t\tau)\\ \sin(t\tau)\end{bmatrix} d\tau \tag{30}$$

where

$$\left.\begin{aligned} t &= \sqrt{\frac{k_t}{d}}\,y \\ \tau &= \sqrt{\frac{k_t}{d}}\,y' \\ T &= \sqrt{\frac{k_t}{d}}\,a \\ \sigma &= \frac{d}{b} \end{aligned}\right\} \tag{31}$$

y, y' = coordinates on the reflectors
$F(\sigma, t)$ = field function for the perturbed modes
P = transfer ratio of the field intensity per reflection.

When $\sigma = 0$, the system is reduced to a Fabry-Perot resonator composed of plane parallel reflectors and the integral equation coincides with that reported by A. G. Fox et al. [14]. The attenuation constant due to diffraction can be obtained from

$$\alpha_d = \frac{k_{mn}}{h_{mn}d}\ln\left|\frac{1}{P}\right|. \tag{32}$$

IV. Example

Calculations are made on the transmission properties for an example of a confocal system with a spacing $d = 4m$, at a wavelength $\lambda = 0.04$ m. Dimensional proportionality is preserved except for the thermal loss for the systems having the same ratio $d/\lambda = 100$.

The transverse wavenumber k_{mn} is obtained from (22).

$$k_{mn} = \frac{\pi}{4}\left(m + \frac{n}{2} + \frac{1}{4}\right). \tag{33}$$

The condition to have a field vanishing in the y-direction is

$$k_{mn} < k_0 = \frac{2\pi}{0.04} \tag{34}$$

then,

$$m + \frac{n}{2} < 200.5. \tag{35}$$

The number of modes that satisfy this condition amounts to 40 000, of which 200 modes belong to the fundamental mode group in the y-direction ($n = 0$). In order to simplify the expression, propagation direction θ of the elementary waves

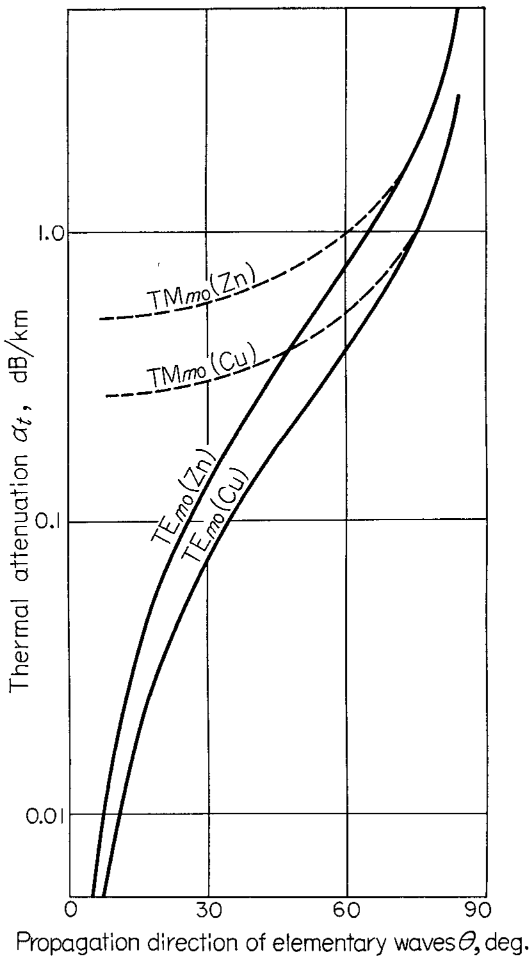

Fig. 6. Thermal loss ($b=d=4$ m, $\lambda=0.04$ m).

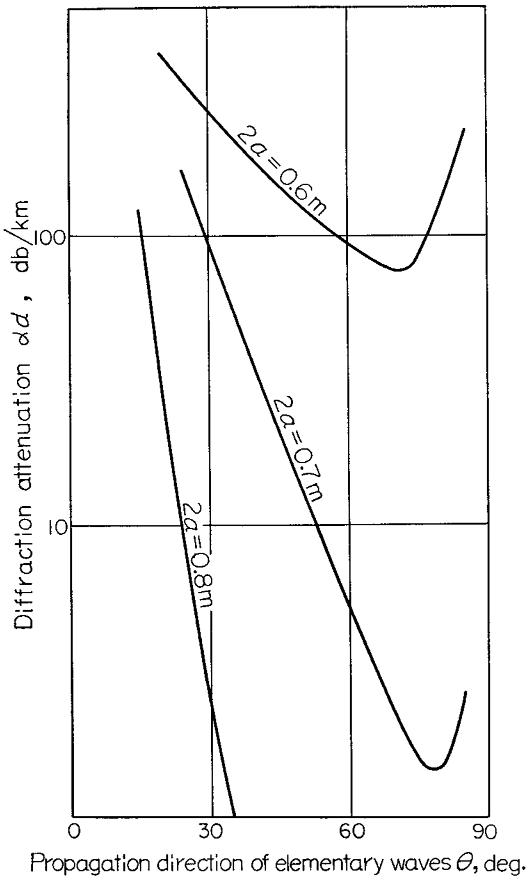

Fig. 7. Diffraction loss ($b=d=4$ m, $\lambda=0.04$ m).

is used as a parameter instead of mode notation in the following treatment. The spot sizes are obtained from (25) and (26) as,

$$w_0 = \frac{0.1595}{\sqrt{\sin\theta}} \tag{36}$$

$$w_s = \frac{0.2255}{\sqrt{\cos\theta}}\,. \tag{37}$$

Figure 6 shows the attenuation constant due to thermal loss for the two kinds of reflector material.

Diffraction attenuation is the same for the TM_{mn} and TE_{mn} modes and is shown in Fig. 7 for three cases of reflector width. When the angle θ gets small, the distance for the elementary beam wave from reflection to reflection tends to infinity. For the large value of the angle, the distance is nearly d, while the number of reflections per unit length increases for the increasing angle. There is an angle which gives the minimum diffraction for the given reflectors.

V. Experiment

Basic measurements have been made on the reflectors shown in Fig. 8. These reflectors are machined from aluminium blocks and supported on confocal condition with iron angles. The aperture width $2a$ of the arc is 0.2 m and the spacing d is 0.5 m. The total width of the reflector is 0.28 m including plane flanges on both sides of the arc, as is shown in the photograph.

Q-measurement was made by varying the length of the reflectors from 0.5 m to 4.5 m to obtain the attenuation constant. Figure 9 shows the theoretical and measured values for the $TE_{19.0}$, $TE_{23.0}$, $TE_{27.0}$, and $TE_{29.0}$ modes at 9.5 GHz. The measured values are less than the theoretical ones. This is supposedly an effect of the plane flanges along both sides of the reflector arc. Figure 10 shows the theoretical and measured values of the guide wavelength of the TE_{m0} modes versus frequency.

VI. Conclusion

A new beam wave transmission system was proposed. The analysis of the system was made on the mode function and the transmission attenuation. The theoretical results proved to be in accord with those from physical observations. Basic properties of the system were measured on a pair of reflectors.

An interesting feature of this transmission is its belt-shaped field distribution between the reflectors as well as a low attenuation. Using this feature, we are intending to develop railway applications of this system. The reflectors are installed on both sides of a railway track. The beam wave transmission plays a role of the transmission medium for the obstacle detection radar or a communication [14].

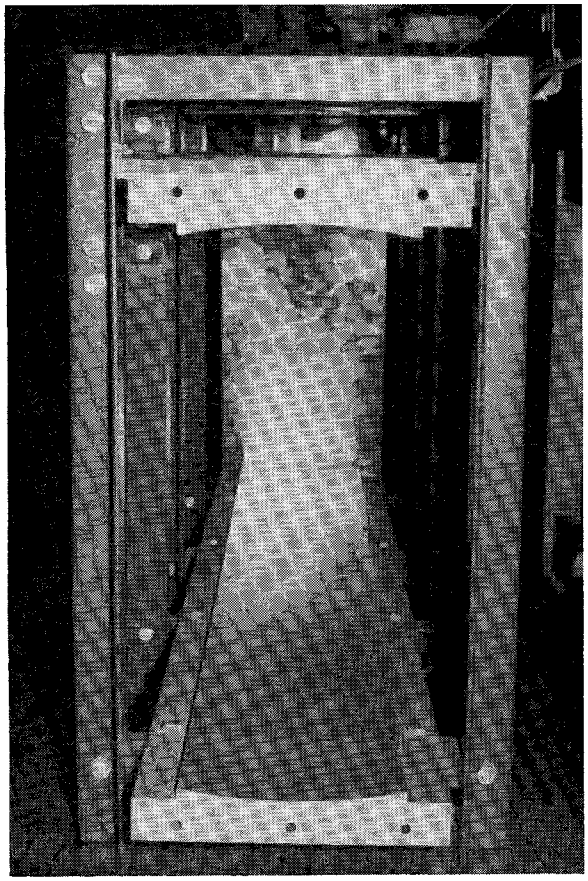

Fig. 8. Reflector for basic experiment.

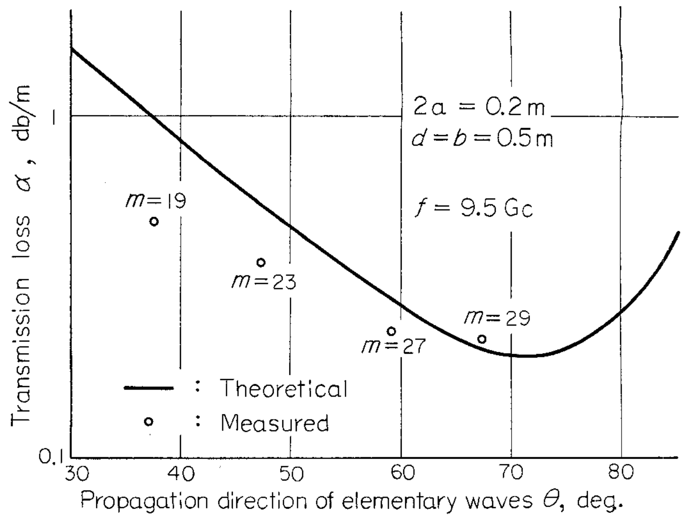

Fig. 9. Measured transmission loss of basic experiment.

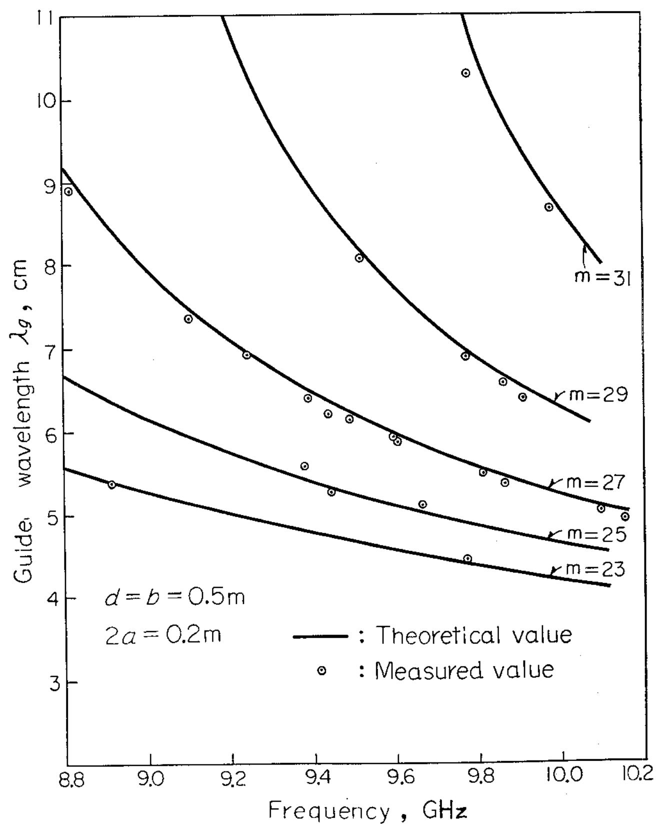

Fig. 10. Measured guide wavelength of the TE_{m0} modes.

Acknowledgment

The authors are indebted to Prof. K. Takiyama of Doshisha University and T. Nagao of Sumitomo Electric Ind., Ltd., for their discussions and contributions to the experiments. The authors would also like to thank the staff of the Technical Research Institute of the Japanese National Railways for their encouragement and suggestions on railway applications.

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